EXERCISE – III HINTS & SOLUTIONS

Sol.1 **(a)** $\sin\theta = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$

$\theta = n\pi + (-1)^n\frac{\pi}{4}$, $n \in I$

(b) $\tan(x-1) = \sqrt{3} = \tan\frac{\pi}{3}$

$(x-1) = n\pi + \frac{\pi}{3}$, $n \in I$

$x = n\pi + \frac{\pi}{3} + 1$, $n \in I$

(c) $\tan\theta = -1 = \tan\left(\frac{-\pi}{4}\right)$

$\theta = n\pi + \left(\frac{-\pi}{4}\right) = n\pi - \frac{\pi}{4}$, $n \in I$

(d) $\operatorname{cosec}\theta = \frac{2}{\sqrt{3}} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}$

$\Rightarrow \theta = n\pi + (-1)^n\frac{\pi}{3}$, $n \in I$

(e) $2\cot^2\theta = \operatorname{cosec}^2\theta$

$\Rightarrow \cot^2\theta = \operatorname{cosec}^2\theta - \cot^2\theta$

$\Rightarrow \cot^2\theta = 1 = \cot^2\frac{\pi}{4} \Rightarrow \theta = n\pi \pm \frac{\pi}{4}$; $n \in I$

Sol.2 $\sin 9\theta = \sin\theta$

$9\theta = 2n\pi + \theta$ or $9\theta = (2n+1)\pi - \theta$, $n \in I$

$\Rightarrow 8\theta = 2n\pi$ or $10\theta = (2n+1)\pi$, $n \in I$

$\Rightarrow \theta = \frac{n\pi}{4}$ or $\theta = (2n+1)\frac{\pi}{10}$; $n \in I$

Sol.3 $\cot\theta + \tan\theta = 2\operatorname{cosec}\theta$

$\Rightarrow \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \frac{2}{\sin\theta}$

$\Rightarrow \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} = \frac{2}{\sin\theta}$ If $\sin\theta \neq 0$ & $\cos\theta \neq 0$

$\Rightarrow \sin\theta - 2\sin\theta\cos\theta = 0$

$\Rightarrow \sin\theta(1 - 2\cos\theta) = 0$

$\Rightarrow$ but $\sin\theta \neq 0$ so $\cos\theta = \frac{1}{2} = \cos\frac{\pi}{3}$

$\theta = 2n\pi \pm \frac{\pi}{3}$; $n \in I$

Sol.4 $\sin 2\theta = \cos 3\theta$

$\Rightarrow \cos\left(\frac{\pi}{2} - 2\theta\right) = \cos 3\theta$

$\Rightarrow 3\theta = 2n\pi \pm \left(\frac{\pi}{2} - 2\theta\right)$

$\Rightarrow 5\theta = 2n\pi + \frac{\pi}{2}$ or $\theta = 2n\pi - \frac{\pi}{2}$

$\Rightarrow \theta = \frac{2n\pi}{5} + \frac{\pi}{10}$ or $\theta = 2n\pi - \frac{\pi}{2}$, $n \in I$

$= \left(2n + \frac{1}{2}\right)\frac{\pi}{5}$n, $\in I$

Sol.5 $\cot\theta = \tan 8\theta$

$\Rightarrow \tan\left(\frac{\pi}{2} - \theta\right) = \tan 8\theta \Rightarrow 8\theta = n\pi + \frac{\pi}{2} - \theta$

$\Rightarrow 9\theta = n\pi + \frac{\pi}{2} \Rightarrow \theta = \frac{n\pi}{9} + \frac{\pi}{18}$; $n \in I$

$\Rightarrow \theta = \left(n + \frac{1}{2}\right)\frac{\pi}{9}$; $n \in I$

Sol.6 $\tan^2\theta - (1+\sqrt{3})\tan\theta + \sqrt{3} = 0$

$(\tan\theta - 1)(\tan\theta - \sqrt{3}) = 0$

$\tan\theta = 1$ or $\tan\theta = \sqrt{3}$

$\theta = n\pi + \frac{\pi}{4}$, $n \in I$ or $\theta = n\pi + \frac{\pi}{3}$; $n \in I$

Sol.7 $\theta \in (0^\circ, 90^\circ)$

$\sec^2\theta \,.\, \operatorname{cosec}^2\theta + 2\operatorname{cosec}^2\theta = 8$

$\Rightarrow \frac{1 + 2\cos^2\theta}{\sin^2\theta\cos^2\theta} = 8$ diff $\sin\theta \neq 0$

$\cos\theta \neq 0$

$\Rightarrow 1 + 2\cos^2\theta = 8(1 - \cos^2\theta)\cos^2\theta$

$\Rightarrow 8\cos^4\theta - 6\cos^2\theta + 1 = 0$

$\Rightarrow (4\cos^2\theta - 1)(2\cos^2\theta - 1) = 0$

$\Rightarrow \cos^2\theta = \left(\frac{1}{2}\right)^2$ or $\cos^2\theta = \left(\frac{1}{\sqrt{2}}\right)^2$

$\Rightarrow \cos\theta = \frac{1}{2}$ or $\cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{3}$ or $\theta = \frac{\pi}{4}$

Sol.8

$4\cos^2\theta - 3 = 2\sin\theta$ ($\because$ $\cos\theta \neq 0$)

$\Rightarrow 4 - 4\sin^2\theta - 3 = 2\sin\theta$

$\Rightarrow 4\sin^2\theta + 2\sin\theta - 1 = 0$

$$\Rightarrow \sin\theta = \frac{-2 \pm \sqrt{4 - 4(4)(-1)}}{8} = \frac{-1 \pm \sqrt{5}}{4}$$

$$\sin\theta = \frac{\sqrt{5}-1}{4} \text{ or } \sin\theta = -\frac{(\sqrt{5}+1)}{4}$$

$$\Rightarrow \sin\theta = \sin\frac{\pi}{10} \text{ or } \sin\theta = -\left(\cos\frac{\pi}{5}\right)$$

$$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{10} \text{ or } \sin\theta = \sin\left(\frac{-3\pi}{10}\right)$$

$$\theta = m\pi + (-1)^n \left(\frac{-3\pi}{10}\right), m \in I$$

Sol.9 $\cot\theta - \tan\theta = 2$

$1 - \tan^2\theta = 2\tan\theta$; $\tan\theta \neq 0$

$\tan^2\theta + 2\tan\theta - 1 = 0$

$$\tan\theta = -\frac{2 \pm \sqrt{4+4}}{2} = -1 \pm \sqrt{2}$$

$\tan\theta = \sqrt{2} - 1$ or $\tan\theta = -(\sqrt{2} + 1)$

$\tan\theta = \tan\frac{\pi}{8}$ $\tan\theta = -\tan\frac{3\pi}{8}$

$\theta = n\pi + \frac{\pi}{8}, n \in I$ or $= \tan\left(\frac{-3\pi}{8}\right)$

$$\theta = \left(2n + \frac{1}{4}\right)\frac{\pi}{2}, n \in I \text{ or } \theta = m\pi + \frac{-3\pi}{8}$$

$$\theta = m\pi - \frac{\pi}{2} + \frac{\pi}{8}$$

$$\theta = (2m-1)\frac{\pi}{2} + \frac{\pi}{8}$$

$$\theta = ((2m-1) + \frac{1}{4})\frac{\pi}{2}, m \in I$$

$$\text{combined } \theta = \left(n + \frac{1}{4}\right)\frac{\pi}{2} \; n \in I$$

Sol.10 $\sin\theta + \sin 3\theta + \sin 5\theta = 0$

$\sin 3\theta + 2\sin 3\theta \cos 2\theta = 0$

$\sin 3\theta (1 + 2\cos 2\theta) = 0$

$\sin 3\theta = 0$ or $1 + 2\cos 2\theta = 0$

$3\theta = n\pi$ $\cos 2\theta = -\frac{1}{2} = \cos\left(\frac{2\pi}{3}\right)$

$\theta = \frac{n\pi}{3}, n \in I$ $2\theta = 2m\pi \pm \left(\frac{2\pi}{3}\right)$

$$\theta = m\pi \pm \frac{\pi}{3} \; m \in I$$

Sol.11 $\cos\theta + \sin\theta = \cos 2\theta + \sin 2\theta$

$$\Rightarrow \frac{1}{\sqrt{2}}(\cos\theta + \sin\theta) = \frac{1}{\sqrt{2}}(\cos 2\theta + \sin 2\theta)$$

$$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \cos\left(2\theta - \frac{\pi}{4}\right)$$

$$\Rightarrow 2n\pi \pm \left(\theta - \frac{\pi}{4}\right) = 2\theta - \frac{\pi}{4}$$

$$\Rightarrow 2\theta - \frac{\pi}{4} = 2n\pi + \theta - \frac{\pi}{4} \Rightarrow \theta = 2n\pi, n \in I$$

$$\& \; 2\theta - \frac{\pi}{4} = 2n\pi - \theta + \frac{\pi}{4}$$

$$\Rightarrow 3\theta = 2n\pi + \frac{\pi}{2} \Rightarrow q = \frac{2n\pi}{3} + \frac{\pi}{6}, n \in I$$

Sol.12 $\theta \in (0, \pi)$

$\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0$

$2\cos 5\theta \cos\theta + 2\cos^2\theta = 0$

$2\cos\theta [\cos 5\theta + \cos\theta]$

$\cos\theta = 0$ or $\cos 2\theta = 0$ or $\cos 3\theta = 0$

$\theta = \frac{\pi}{2}$ or $\theta = \frac{\pi}{4}$ or $3\theta = (2k+1)\frac{\pi}{2}$

$\theta = \frac{\pi}{4} = \frac{3\pi}{4}$ or $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$$\theta = \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{6}$$

Sol.13 $\cos^2 x + \cos^2 2x + \cos^2 3x = 1$

$$\Rightarrow \frac{1+\cos 2x}{2} + \frac{1+\cos 4x}{2} + \frac{1+\cos 6x}{2} = 1$$

$\Rightarrow \cos 2x + \cos 4x + \cos 6x + 3 = 2$

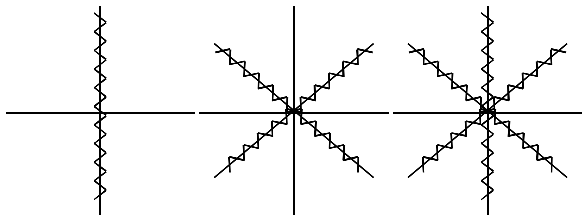
$\Rightarrow \cos 2x + \cos 4x + \cos 6x + 1 = 0$

Previous Q.

$x = (2m+1)\dfrac{\pi}{2}$, $2x = (2n+1)\dfrac{\pi}{2}$, $3x = (2k+1)\dfrac{\pi}{2}$, $k \in I$

$m \in I$ $\quad x = (2n+1)\dfrac{\pi}{4}$, $x = (2k+1)\dfrac{\pi}{6}$, $k \in I$

verticle line $\quad n \in I$



vertical line is induced
so remaining sol.

$x = k\pi \pm \dfrac{\pi}{6} \qquad k \in I$

sol. are

or $x = (2m+1)\dfrac{\pi}{2}$ $m \in I$, $x = (2n+1)\dfrac{\pi}{4}$;

$n \in I$, $x = \left(k\pi \pm \dfrac{\pi}{6}\right)$, $k \in I$

Sol.14 $\sin^2\theta - \sin^2(n-1)\theta = \sin^2\theta$, n is constant & $n \neq 0,1$

$\Rightarrow \sin(n\theta + n\theta - \theta)\sin(n\theta - n\theta + \theta) = \sin^2\theta$

$\Rightarrow \sin(2n\theta - \theta)\sin\theta = \sin^2\theta$

$\Rightarrow \sin\theta = 0$ or $\sin(2n-1)\theta - \sin\theta = 0$

$\theta = m\pi$, $m \in I$ or $2\cos\left(\dfrac{2n\theta - \theta + \theta}{2}\right)\sin\left(\dfrac{2n\theta - \theta - \theta}{2}\right) = 0$

$2\cos(n\theta)\sin(n-1)\theta = 0$

$\cos n\theta = 0$ or $\sin(n-1) = 0$

$n\theta = (2k+1)\dfrac{\pi}{2}$ or $(n-1)\theta = p\pi \quad k, p, \in I$

$\theta = \dfrac{(2k+1)}{2}\dfrac{\pi}{n}$ or $\theta = \dfrac{p\pi}{(n-1)}$; $p \in I$

Sol.15 $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$

$\Rightarrow \dfrac{\sqrt{3}}{2}\sin\theta - \dfrac{1}{2}\cos\theta = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(\theta - \dfrac{\pi}{6}\right) = \dfrac{1}{\sqrt{2}} = \sin\dfrac{\pi}{4}$

$\Rightarrow \theta - \dfrac{\pi}{6} = n\pi + (-1)^n\dfrac{\pi}{4}$

$\Rightarrow \theta = n\pi + \dfrac{\pi}{6} + (-1)^n\dfrac{\pi}{4}$; $n \in I$

Sol.16 $\text{cosec}\theta = \cot\theta + \sqrt{3}$

$\Rightarrow \text{cosec}^2\theta = \cot^2\theta + 2\sqrt{3}\cot\theta + 3$

$\Rightarrow \text{cosec}^2\theta - \cot^2\theta = 2\sqrt{3}\cot\theta + 3$

$\Rightarrow 1 = 2\sqrt{3}\cot\theta + 3 \qquad \Rightarrow \cot\theta = -\dfrac{2}{2\sqrt{3}}$

$\Rightarrow \tan\theta = -\sqrt{3} \qquad \Rightarrow \tan\theta = \tan\left(-\dfrac{\pi}{3}\right)$

$\theta = n\pi + \left(-\dfrac{\pi}{3}\right)$

$\theta = n\pi - \dfrac{\pi}{3}$, $n \in I$

but IV quadrant is reject

Aliter :

$\dfrac{1}{\sin\theta} = \dfrac{\cos\theta}{\sin\theta} + \sqrt{3}$, $\sin\theta \neq 0$

$\sqrt{3}\sin\theta + \cos\theta = 1$

$\dfrac{\sqrt{3}}{2}\sin\theta + \dfrac{1}{2}\cos\theta = \dfrac{1}{2}$

$\cos\left(\theta - \dfrac{\pi}{3}\right) = \dfrac{1}{2}$

$\theta - \dfrac{\pi}{3} = 2m\pi \pm \dfrac{\pi}{3}$, $m \in I$

$\theta = 2m\pi$ or $\theta = 2m\pi + \dfrac{2\pi}{3} \qquad m \in I$

reject cosecθ & cotθ is present. because does not satisfy given equation (in IV quadrant)

Sol.17 $5\sin\theta + 2\cos\theta = 5$

$\Rightarrow \dfrac{10\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} + \dfrac{2(1-\tan^2\frac{\theta}{2})}{1+\tan^2\frac{\theta}{2}} = 5$

$\Rightarrow 10\tan\dfrac{\theta}{2} + 2 - 2\tan^2\dfrac{\theta}{2} = 5 + 5\tan^2\dfrac{\theta}{2}$

$\Rightarrow 7\tan^2\dfrac{\theta}{2} - 10\tan\dfrac{\theta}{2} + 3 = 0$

$\Rightarrow \left(\tan\dfrac{\theta}{2} - 1\right)\left(7\tan\dfrac{\theta}{2} - 3\right) = 0$

$\Rightarrow \frac{\theta}{2} = n\pi + \frac{\pi}{4}$ or $\frac{\theta}{2} = n\pi + \alpha$

$\Rightarrow \theta = 2n\pi + \frac{\pi}{2}$, $n \in I$

or $\theta = 2n\pi + 2\alpha$; $\alpha = \tan^{-1}\frac{3}{7}$, $n \in I$

Sol.18 $\tan 2\theta \tan\theta = 1$

$\tan 2\theta = \cot\theta$

$\Rightarrow \tan 2\theta = \tan\left(\frac{\pi}{2} - \theta\right) \Rightarrow 2\theta = n\pi + \frac{\pi}{2} - \theta$

$\Rightarrow 3\theta = n\pi + \frac{\pi}{2} \Rightarrow 3\theta = n\pi + \frac{\pi}{2}$

$\Rightarrow \theta = (2n+1)\frac{\pi}{6} n \in I$

$2n + 1 \neq 3k$

Aliter : $1 - \tan 2\theta \tan\theta = 0$

$\because \tan 3\theta =$ not. defined

$3\theta = n\pi + \frac{\pi}{2} \Rightarrow \theta = (2n+1)\frac{\pi}{6} \quad n \in I$

Alter: $\frac{2\tan\theta}{1-\tan^2\theta}\tan\theta = 1$

$2\tan^2\theta = 1 - \tan^2\theta$

$\tan^2\theta = \frac{1}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{6}$

Sol.19 $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$

$\Rightarrow \tan\theta + \tan 2\theta = \sqrt{3}(1 - \tan\theta\tan 2\theta)$

$\Rightarrow \frac{\tan\theta + \tan 2\theta}{(1 - \tan\theta\tan 2\theta)} = \sqrt{3} \ \{\because 1 - \tan 2\theta\tan\theta \neq 0\}$

$\Rightarrow \tan 3\theta = \sqrt{3} \Rightarrow 3\theta = n\pi = \frac{\pi}{3}$

$\Rightarrow \theta = \left(n + \frac{1}{3}\right)\frac{\pi}{3}$; $n \in I$

Sol.20 $\tan x . \tan\left(x + \frac{\pi}{3}\right)$; $\tan\left(x + \frac{2\pi}{3}\right) = \sqrt{3}$

$\Rightarrow \tan x . \tan\left(\frac{\pi}{3} + x\right) \tan\left[\pi - \left(\frac{\pi}{3} - x\right)\right] = \sqrt{3}$

$\Rightarrow \tan x \tan x\left(\frac{\pi}{3} + x\right) \tan\left(\frac{\pi}{3} - x\right) = -\sqrt{3}$

$\Rightarrow \tan 3x = -\sqrt{3} = \tan\left(-\frac{\pi}{3}\right) \Rightarrow 3x = n\pi - \frac{\pi}{3}$

$\Rightarrow x = \frac{n\pi}{3} - \frac{\pi}{9}$, $n \in I$

Sol.21 $\tan\theta + \sin\phi = \frac{3}{2}$; $\tan^2\theta + \cos^2\phi = \frac{7}{4}$

$\Rightarrow \tan^2\theta + 1 - \sin^2\phi = \frac{7}{4}$

$\Rightarrow \tan^2\theta - \sin^2\phi = \frac{3}{4}$

$\Rightarrow (\tan\theta + \sin\phi)(\tan\theta - \sin\phi) = \frac{3}{4}$

$\Rightarrow \frac{3}{2}(\tan\theta - \sin\phi) = \frac{3}{4}$

$\Rightarrow \tan\theta - \sin\phi = \frac{1}{2}$

$\tan\theta + \sin\phi = \frac{3}{2}$

$2\tan\theta = 2$

$\tan\theta = 1$

$\theta = n\pi + \frac{\pi}{4}$, $n \in \pi$

$2\sin\phi = 1$

$\sin\phi = \frac{1}{2}$

$\phi = n\pi + (-1)^n\frac{\pi}{6}$, $n \in I$

Sol.22 $a\tan\theta + b\sec\theta = c$

$b\sec\theta = c - a\tan\theta$

$\Rightarrow \quad b^2\sec^2\theta = c^2 + a^2\tan^2\theta - 2ac\tan\theta$

$\Rightarrow \quad b^2 + b^2\tan\theta = c^2 + a^2\tan\theta - 2ac\tan\theta$

$\Rightarrow (a^2 - b^2)\tan^2\theta - 2ac\tan\theta + c^2 - b^2 = 0 \begin{cases}\tan\alpha \\ \tan\beta\end{cases}$

$\tan\alpha + \tan\beta = \frac{2ac}{a^2 - b^2}$ & $\tan\alpha . \tan\beta = \frac{c^2 - b^2}{a^2 - b^2}$

$\Rightarrow \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{2ac/a^2 - b^2}{1 - (c^2 - b^2)/(a^2 - b^2)}$

$= \frac{2ac}{a^2 - b^2 - c^2 + b^2} = \frac{2ac}{a^2 - c^2}$

Sol.23 $a \cos 2\theta + b \sin 2\theta = c$

$\Rightarrow b \ \sin 2\theta = c - a \cos 2\theta$

$\Rightarrow b^2 \sin^2 2\theta = c^2 + a^2\cos^2 2\theta - 2ac \cos 2\theta$

$\Rightarrow b^2 - b^2 \cos^2 2\theta = c^2 + a^2 \cos^2 2\theta - 2ac \cos 2\theta$

$\Rightarrow (a^2 + b^2) \cos^2 2\theta - 2ac \cos 2\theta$

$+ (c^2 - b^2) = 0 \ \Big\langle$ cos, cos

$$\cos 2\alpha + \cos 2\beta = \frac{2ac}{a^2+b^2} \ \& \ \cos 2\alpha \,.\, \cos 2\beta = \frac{c^2-b^2}{a^2+b^2}$$

$$2\cos^2\alpha - 1 + 2\cos^2\beta - 1 = \frac{2ac}{a^2+b^2}$$

$$\cos^2\alpha + \cos^2\beta = \frac{1}{2}\left[\frac{2ac}{a^2+b^2}+2\right]$$

$$= \frac{a^2+ac+b^2}{a^2+b^2}$$

Sol.24 $(\sin 2\theta + \sqrt{3} \ \cos 2\theta)^2 - 5 = \cos\left(\frac{\pi}{6}-2\theta\right)$

$$\Rightarrow \left[2\left\{\frac{\sqrt{3}}{2}\cos 2\theta + \frac{1}{2}\sin 2\theta\right\}\right]^2 - 5 = \cos\left(\frac{\pi}{6}-2\theta\right)$$

$\Rightarrow 4t^2 - 5 = t \Rightarrow \ 4t^2 - t - 5 = 0$

$\Rightarrow (t + 1)(4t - 5) = 0 \qquad \{\text{let } \cos\left(\frac{\pi}{6}-2\theta\right) = t$

$\Rightarrow \cos\left(\frac{-\pi}{6}+2\theta\right) = -1$ or $\cos\left(\frac{\pi}{6}-2\theta\right) \neq \frac{5}{4}$

$\Rightarrow -\left(\frac{\pi}{6}-2\theta\right) = 2n\pi + \pi \Rightarrow 2\theta = 2n\pi + \pi + \frac{\pi}{6}$

$\Rightarrow \theta = \frac{7\pi}{12}, \qquad n \in I, \ \theta \in [0, 2\pi]$

at $n = 0 \Rightarrow \theta = \frac{7\pi}{12}$ & at $n = 1 \Rightarrow \theta = \frac{19\pi}{12}$

Sol.25 $1 + 2\,\text{cosec}x = -\dfrac{\sec^2\frac{x}{2}}{2}$ $(\text{cosec } x \neq 0 \Rightarrow x \notin n\pi)$

$$\Rightarrow \frac{\sin x + 2}{\sin x} = \frac{-1}{2\cos^2\frac{x}{2}}$$

$\Rightarrow (\sin x + 2)(1 + \cos x) + \sin x = 0$

$\Rightarrow 2\sin x + 2 \cos x + \sin x \cos x + 2 = 0$

$\Rightarrow 4 (\sin x + \cos x) + (\sin^2 x + 1) + 3 = 0$

$\Rightarrow 4 (\sin x + \cos x) + (\sin x + \cos x)^2 + 3 = 0$

Let $\sin x + \cos x = t$

$\Rightarrow 4t + t^2 + 3 = 0 \qquad \Rightarrow t^2 + 4t + 3 = 0$

$\Rightarrow (t + 1)(t + 3) = 0 \qquad \Rightarrow \sin x + \cos x = -1$

or $\qquad \sin x + \cos x \neq -3 - \sqrt{2} \le t \le \sqrt{2}$

$\Rightarrow \qquad \cos\left(x - \frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

$x - \frac{\pi}{4} = 2n\pi \pm \frac{3\pi}{4} \qquad n \in I$

$\Rightarrow \qquad x = 2n\pi + \pi \qquad x = 2n\pi - \frac{\pi}{2} \quad n \in I$

$\Rightarrow \qquad x = (2n + 1)\pi \quad n \in I$

But $x \notin n\pi$

Sol.26 $2\sin x = 3x^2 + 2x + 3$

In R.H.S. $\qquad D = 4 - 36 = -32$

$\therefore \qquad$ min value of RHS $= \frac{-D}{4.3} = \frac{8}{3}$

Max. value of L.H.S. = 2

L.H.S. $\neq$ R.H.S. $\Rightarrow$ No solution

$\forall x \in R$

$x \in \phi$

Sol.27 $2 + 7 \tan^2\theta = 3.25 \sec^2\theta \ 0^\circ < \theta < 360^\circ$

$\Rightarrow 7(1 + \tan^2\theta) - 5 = \frac{325}{100} \sec^2\theta$

$\Rightarrow 7 \sec^2\theta - 5 = \frac{13}{4} \sec^2\theta$

$\Rightarrow 28 \sec^2\theta - 20 = 13 \sec^2\theta \quad \Rightarrow 15 \sec^2\theta = 20$

$\Rightarrow \sec^2\theta = \frac{4}{3}$

$\theta = 2n\pi \pm \frac{\pi}{6}$

$\Rightarrow \theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$

Sol.28 $5^{1/2} + 5^{\frac{1}{2}+\log_5(\sin x)} = 15^{\frac{1}{2}+\log_{15}\cos x}$

$\Rightarrow \sqrt{5} + \sqrt{5} \cdot 5^{\log_5 \sin x} = \sqrt{3}\ \sqrt{5} = 15^{\log_{15}\cos x}$

$\sin x > 0$ & $\cos x > 0$

$\Rightarrow \qquad x$ in Ist quadrant

$\Rightarrow \qquad 1 + \sin x = \sqrt{3} \ \cos x$

$\Rightarrow \qquad \frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x = \frac{1}{2}$

$\Rightarrow \cos\left(x+\frac{\pi}{6}\right) = \cos\frac{\pi}{3}$ or $x + \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{3}$

$\Rightarrow x = 2n\pi + \frac{\pi}{6}$ or $x = 2n\pi - \frac{\pi}{2}$ (reject)

In general, $x = 2n\pi + \frac{\pi}{6}$, $n \in I$

Sol.29 $\sin\theta + \sin 5\theta = \sin 3\theta \qquad 0 \le \theta \le \pi$

$\Rightarrow 2\sin 3\theta \cos 2\theta = \sin 3\theta$

$\Rightarrow \sin 3\theta\,(2\cos 2\theta - 1) = 0$

$\Rightarrow \sin 3\theta = 0 \quad$ or $\quad \cos\theta = \frac{1}{2}$

$\Rightarrow \sin 3\theta = 0 \quad$ or $\quad 2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$

$\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi \qquad \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6} \notin [0, \pi]$

$\Rightarrow \theta = 0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$ & π

Sol.30 $2\sin 11x + \cos 3x + \sqrt{3}\sin 3x = 0$

$\Rightarrow \frac{1}{2}\cos 3x + \frac{\sqrt{3}}{2}\sin 3x = -\sin 11x$

$\Rightarrow \sin\left(3x+\frac{\pi}{6}\right) = \sin(-11x)$

$\Rightarrow 3x + \frac{\pi}{6} = 2n\pi + (-11x)$

or $3x + \frac{\pi}{6} = (2n+1)\pi - (-11)x$

$\Rightarrow 14x = 2n\pi - \frac{\pi}{6} \qquad -8x + \frac{\pi}{6} = 2n\pi + \pi$

$\Rightarrow x = \frac{n\pi}{7} - \frac{\pi}{84}, n \in I \quad -8x = 2n\pi + \frac{5\pi}{6}$

$x = \frac{n\pi}{4} - \frac{5\pi}{48}, n \in I$

Sol.31 $\cos\theta \,.\, \cos 2\theta \,.\, \cos 3\theta = \frac{1}{4}$

$\Rightarrow \quad 2\cos 2\theta \,.\, 2\cos\theta\cos 3\theta = 1$

$\Rightarrow \quad 2\cos 2\theta\,[\cos 4\theta + \cos 2\theta] = 1$

$\Rightarrow \quad 2\cos 4\theta\cos 2\theta + (2\cos^2 2\theta - 1) = 0$

$\Rightarrow \quad 2\cos 4\theta\cos 2\theta + \cos 4\theta = 0$

$\Rightarrow \quad \cos 4\theta\,[2\cos 2\theta + 1] = 0$

$\Rightarrow \quad \cos 4\theta = 0$ or $\cos 2\theta = -\frac{1}{2}$

$\Rightarrow \quad 4\theta = 2n\pi$ or $2\theta = 2n\pi \pm \frac{2\pi}{3}$

$\theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8} \qquad \theta = \frac{\pi}{3}, \frac{2\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{\pi}{3}, \frac{5\pi}{8}, \frac{2\pi}{3}, \frac{7\pi}{8}$

Sol.32 $2 + \tan x \,.\, \cot\frac{x}{2} + \cot x \,.\, \tan\frac{x}{2} = 0$

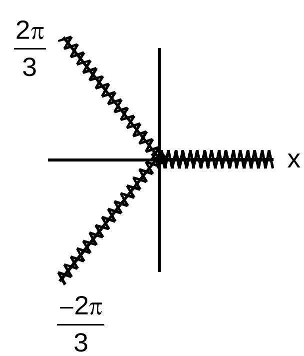
$$\Rightarrow 2 + \frac{\tan x}{\tan\frac{x}{2}} + \frac{\tan\frac{x}{2}}{\tan x} = 0 \qquad \begin{cases} \frac{x}{2}x \ne 2n\pi \\ \frac{x}{2}, x \ne (2n+1)\frac{\pi}{2} \end{cases}$$

$$\Rightarrow \frac{2\tan\frac{x}{2}\tan x + \tan^2 x + \tan^2\frac{x}{2}}{\tan\frac{x}{2}\tan x} = 0 \qquad \begin{cases} \tan\frac{x}{2} \ne 0 \\ \tan x \ne 0 \end{cases}$$

$\Rightarrow \left(\tan x + \tan x\frac{x}{2}\right)^2 = 0 \Rightarrow \tan x = -\tan\frac{x}{2}$

$\Rightarrow \tan x = \tan\left(-\frac{x}{2}\right) \Rightarrow x = n\pi + \left(-\frac{x}{2}\right)$

$\Rightarrow \frac{3x}{2} = n\pi \Rightarrow x = \frac{2n\pi}{3}$



$\therefore$ reject all sol. at $2n\pi$

$x = 2n\pi \pm \frac{2\pi}{3}$, $n \in I$

Sol.33 $\sqrt{13-18\tan x} = 6\tan x - 3 \qquad -2\pi < x < 2\pi$

$\Rightarrow \quad 13 - 18\tan x = 9(4\tan^2 x - 4\tan x + 1)$

$\Rightarrow \quad 13 - 18\tan x = 36\tan^2 x - 36\tan x + 9$

$\Rightarrow \quad 36\tan^2 x - 18\tan x - 4 = 0$

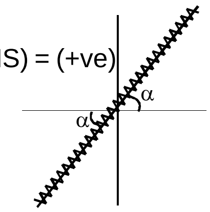
$\Rightarrow \quad 18\tan^2 x - 9\tan x - 2 = 0$

$\Rightarrow \quad (3\tan x - 2)(6\tan x + 1) = 0$

$\Rightarrow \quad \tan x = -\frac{1}{6}$ or $\tan x = \frac{2}{3}$

$\Rightarrow$ R.H.S. = (–ve)

Not possible $\because$ (LHS) = (+ve)



$\tan x = \frac{2}{3} = \tan\alpha$

$x = n\pi + \alpha$

$x = \alpha,\ \pi + \alpha,\ -2\pi + \alpha,\ -\pi + \alpha$

where $\tan\alpha = \frac{2}{3}$

Sol.34 $\sqrt{\cot 3x + \sin^2 x - \frac{1}{4}} + \sqrt{\sqrt{3}\cos x + \sin x - 2}$

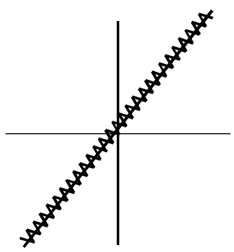
$= \frac{\sin 3x}{2} - \frac{\sqrt{2}}{2}$

$\sqrt{3}\cos x + \sin x \ge 2$

but $\sqrt{3}\cos x + \sin x \in [-2, 2]$

$\therefore \quad \sqrt{3}\cos x + \sin x = 2$

$\Rightarrow \cos\left(x - \frac{\pi}{6}\right) = 1$



$x - \frac{\pi}{6} = 0 \qquad \Rightarrow x = \frac{\pi}{6}$

check

$\sqrt{0 + \frac{1}{4} - \frac{1}{4}} + \sqrt{2-2} = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$

satisfy at $x = \frac{\pi}{6}$

Sol.35 $\sqrt{1+\sin 2x} - \sqrt{2}\cos 3x = 0$

$\Rightarrow \quad 1 + \sin 2x = 2\cos^2 3x$

$\Rightarrow \quad 1 + \sin 2x = 1 + \cos 6x$

$\Rightarrow \quad \cos 6x = \cos\left(\frac{\pi}{2} - 2x\right)$

$\Rightarrow \quad \cos 6x = \cos\left(\frac{\pi}{2} - 2x\right)$

$\Rightarrow \quad 6x = 2n\pi \pm \left(\frac{\pi}{2} - 2x\right)$

$\Rightarrow 6x = 2n\pi + \frac{\pi}{2} - 2x$ or $6x = 2n\pi - \frac{\pi}{2} + 2x$

$\Rightarrow 8x = 2n\pi + \frac{\pi}{2}$ or $4x = 2n\pi - \frac{\pi}{2}$

$\Rightarrow x = \frac{n\pi}{4} + \frac{\pi}{16}$ or $x = \frac{n\pi}{2} - \frac{\pi}{8}$

$\min\left[\frac{\pi}{16}, \frac{3\pi}{8}\right] = \frac{\pi}{16}$

Sol.36 $\sqrt{3}\sin 2A = \sin 2B$

$\Rightarrow 3\sin^2 2A = \sin^2 2B$

$\Rightarrow 3 - 3\cos^2 2A = 1 - \cos^2 2B$

$\Rightarrow (\sqrt{3}\cos 2A)^2 - (\cos 2B)^2 = 2$

$\Rightarrow (\sqrt{3}\cos 2A)^2 - (\cos 2B)^2 = 2$

$\Rightarrow (\sqrt{3}\cos 2A + \cos 2B)(\sqrt{3}\cos 2A - \cos 2B) = 2$ ... (i)

& $\sqrt{3}\sin^2 A + \sin^2 B = \frac{\sqrt{3}-1}{2}$

$\Rightarrow \sqrt{3}(1 - \cos 2A) + (1 - \cos 2B) = \sqrt{3} - 1$

$\Rightarrow -\sqrt{3}\cos 2A - \cos 2B = -2$

$\Rightarrow \sqrt{3}\cos 2A + \cos 2B = 2$ ....(ii)

by (i) & (ii) $\Rightarrow \sqrt{3}\cos 2A - \cos 2B = 1$ ...(iii)

by (ii) & (iii) $\Rightarrow \cos 2A = \frac{\sqrt{3}}{2} \Rightarrow 2A = 30^\circ \Rightarrow A = 15^\circ$

& $\cos 2B = \frac{1}{2} \Rightarrow 2B = 60^\circ \Rightarrow B = 30^\circ$